

Nomogram-based Optimal Tuning of Ideal PI-Controllers for Second-Order-Like Overdamped Processes

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Abstract— The tuning of an ideal PI-controller used in cascade with a second-order-like process using a new nomogram-based technique is discussed and evaluated. The new technique is based on minimizing an ISTSE error function and presenting the tuned parameters of the controller in a graphical form. The nomograms present also the maximum percentage overshoot and settling time of the time response of the closed-loop control system for a reference unit step input. The effectiveness of the new technique is examined through comparison with another tuning technique.

Index Terms— PI controllers, Controller tuning, Nomogram-based tuning, Second-order-like overdamped processes

I. INTRODUCTION

The first generation of PID controllers is still in use and find a room for application in some industrial applications. The researchers still studying the tuning of the PID controllers through the application of different techniques to simplify the tuning process and simplify the application for the practicing engineers. For this purpose comes this paper to introduce a new approach for the tuning of PI controllers.

O'Dwyer (2001) discussed the design of PI and PID controller tuning rules for processes with delay. His rules allowed the achievement of constant gain and phase margins as the delay varies [1]. Basilio and Matos (2002) proposed tuning methods for PI and PID controllers using parameters from the plant step response. They adjusted the proportional gain to have a step response without overshoot [2]. Tavakoli and Fleming (2003) presented an optimal method for tuning PI controllers for first order plus dead time processes. They minimized the integral of absolute error (IAE) for minimum gain margin of 2 and a minimum phase margin of 60 degrees [3]. Zhao and Collins (2004) applied two advanced PI controller tuning methods to an industrial weigh belt feeder with significant nonlinearities. They used the unfalsified control and the fuzzy control techniques [4].

Liu (2007) presented the work of modelling and simulation of a self-tuning PI control for induction motors. He showed that with the self-tuned PI controller, the design approach was robust to the variations of the induction motor parameters [5]. Mudi, Dey and Lee (2008) presented an improved auto-tuning

system for Ziegler-Nichols tuned PI controllers. They tested the proposed controller for a number of high order linear and nonlinear dead-time processes under both set-point and load disturbance [6]. Nour (2008) proposed an online self-tuning scheme using fuzzy logic controller. He tested the performance of the proposed controller through a wide range of motor speeds as well as with load and parameters variation through simulation [7]. O' Dwyer (2009) published a book collecting the PI and PID controller tuning rules. His work covered delay model, delay model with zero, FOLPD model, FOLPD model with zero, SOSP model, SOSP model with zero, TOSPD model, 5th-order system plus delay model, general model, non-model specific and unstable models [8].

Dubonjic, Nedic, Filipovic and Prsic (2013) proposed a procedure for the design of PI controllers for hydraulic systems with long transmission lines. They presented the algorithm of software procedure for the design of the controller [9]. Pal and Naskar (2013) proposed an intelligent control strategy applied to a real time water pressure control system. They developed an intelligent control scheme by integrating a self-tuning scheme with fuzzy PI controller [10]. Ramadevi and Vijayan (2014) presented a review of various methods for the design of controllers for multiple-input multiple-output system. They used IAE and ISE criteria with various tuning methods such as direct synthesis, sequential relay with ZN settings and IMC based method [11]. Kim, Chung and Moon (2015) presented a method for PI controller tuning used with a PMSG wind turbine to improve its control performance. They used the particle swarm optimization to adjust the PI controller parameters [12]. Ofori, Kamau, Ndera and Muhia (2016) presented a method of obtaining optimal gains for a fuzzy PI controller to control the frequency of Micro Hydro Power Plant. They obtained the optimal PI controller by using of the Bacterial Foraging Algorithm in MATLAB/SIMULINK environment [13]. Nagarajan et. Al. (2016) described the implementation of PI controller for a boost converter in photo voltaic system. They calculated the PI controller parameters using a trial and error method [14].



II. THE PROCESS

The process is a second-order-like-overdamped process. A lot of processes with non-oscillating step response can be approximated to a second-order process with or without time delay. A second-order process has a standard transfer function, $G_p(s)$ given by:

$$G_p(s) = \omega_n^2 / (s^2 + 2\zeta \omega_n s + \omega_n^2) \quad (1)$$

Where:

ω_n = process natural frequency (rad/s)

and ζ = process damping ratio

III. THE PI CONTROLLER

The PI controller is one of the controller-versions of the first generation family of the PID controllers where researchers paid a lot of effort to tune [15]. A PI controller has two gain parameters:

- The proportional gain, K_{pc} .
- The integral gain, K_i .

The transfer function of an ideal PI controller, $G_c(s)$ is:

$$G_c(s) = K_{pc} + K_i/s \quad (2)$$

IV. BLOCK DIAGRAM

The block diagram of the control system incorporating the PI controller and a second-order process is shown in Fig.1.

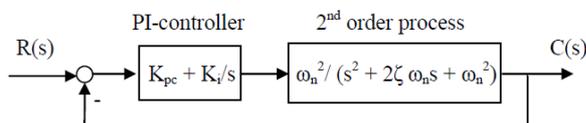


Fig.1 System block diagram.

The control system incorporating the ideal PI controller and a second order process (using the block diagram of Fig.1) has a closed-loop transfer function, $M(s)$ given by:

$$M(s) = (\omega_n^2 K_{pc} s + \omega_n^2 K_i) / \{s^3 + 2\zeta \omega_n s^2 + (\omega_n^2 K_{pc} + \omega_n^2) s + \omega_n^2 K_i\} \quad (3)$$

Eq.3 depicts the fact that the control system comprising an ideal PI controller and a second order process has a third-order transfer function. Therefore it will be subjected to possibility of instability depending of the controller parameters. The Routh-Hurwitz criterion of linear control system stability [16] determines the stability condition of the control system in hand as:

$$K_i < 2\zeta \omega_n (K_{pc} + 1)$$

V. PI CONTROLLER TUNING

The PI controller used to control the second order process is tuned as follows:

1. The step response of the control system to a unit step

input, $c(t)$ is generated using the 'step' command of the MATLAB.

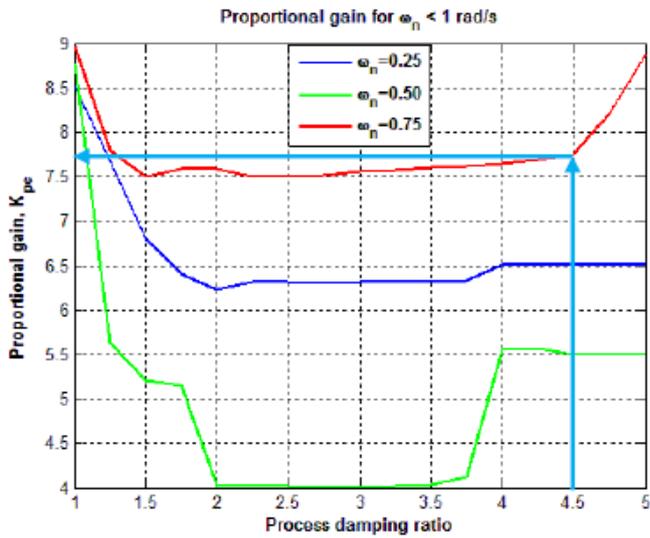
2. An error function, $e(t)$ is defined as the difference between the time response $c(t)$ and the steady state response of the control system which is one in this case.
3. An objective function required by an optimization approach, f is defined as the sum of the time square multiplied by the error square (ISTSE objective function) [17].
4. This objective function is minimized using the MATLAB optimization command 'fminunc' [18].
5. A MATLAB code is written to minimize the ISTSE objective function for a specific natural frequency and damping ratio of the process.
6. The output of the previous step is the optimal parameters of the PI controller for a minimum objective function.
7. The damping ratio of the process is changed within the range: $1 \leq \zeta \leq 5$.
8. The natural frequency of the process is changed within the range: $0.25 \leq \omega_n \leq 2.50$ rad/s.
9. The results are presented in a form of nomograms relating the controller parameters K_{pc} and K_i and the performance parameters (maximum percentage overshoot and settling time for set-point change) to the process parameters.
10. This technique was invented and applied by the author in the field of mechanism synthesis and presented great simplification of complex planar mechanisms synthesis [19] to [21].

VI. PI TUNING NOMOGRAMS

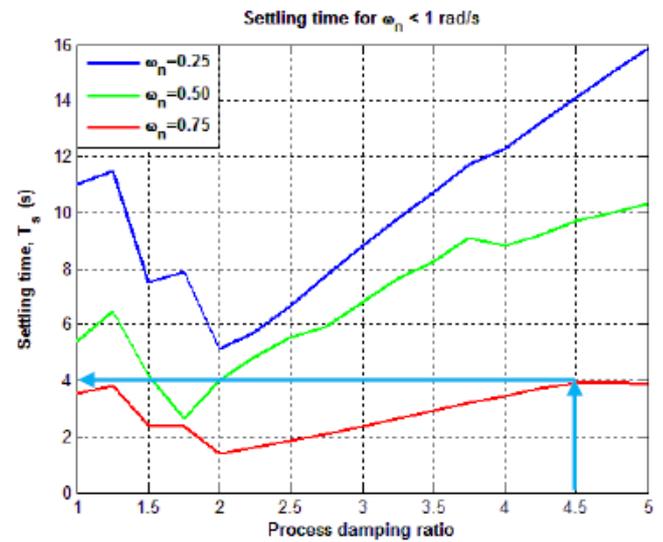
The tuning nomograms are designed for two ranges of process natural frequency as follows:

- From 0.25 to 0.75 rad/s in an increment of 0.25 rad/s.
- From 1 to 2.5 rad/s in an increment of 0.25 rad/s.

The nomograms for the first process natural frequency range are shown in Figs.2(a) through Fig.2(d).

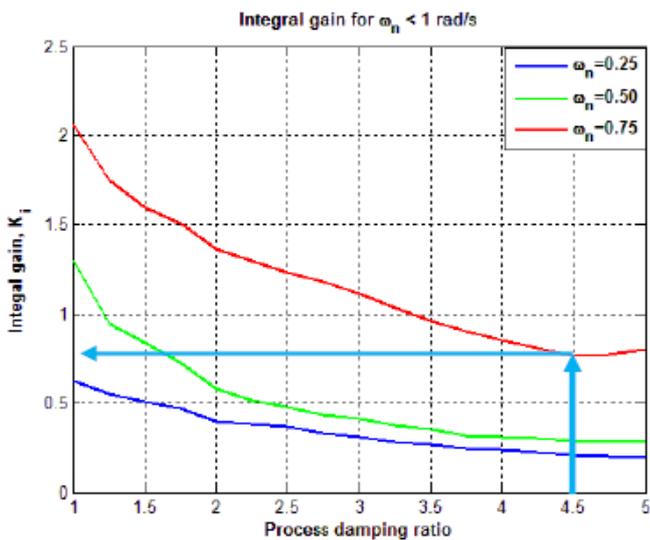


(a)

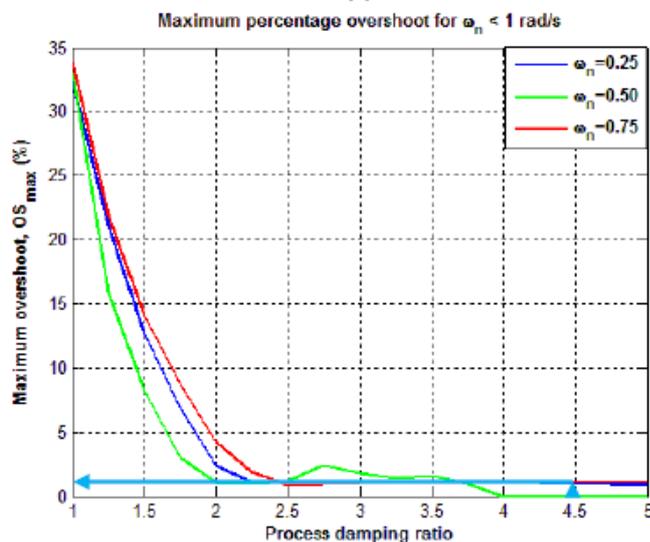


(d)

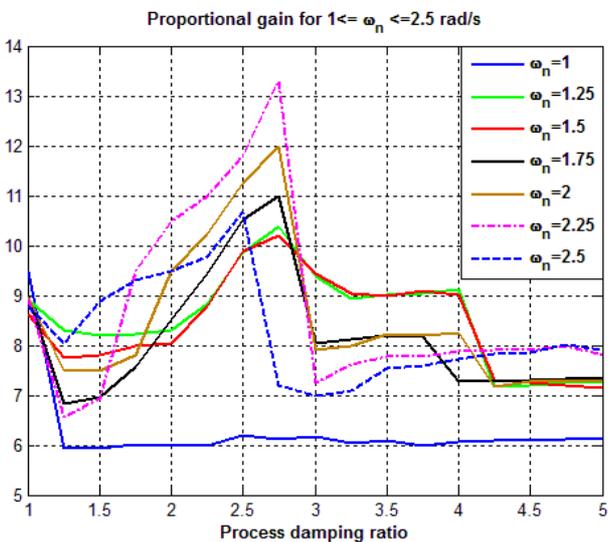
Fig.2 Tuning nomograms for $0.25 \leq \omega_n \leq 0.75$ rad/s.



(b)



(c)



(a)

The nomograms for the second process natural frequency range are shown in Figs.3(a) through Fig.3(d).

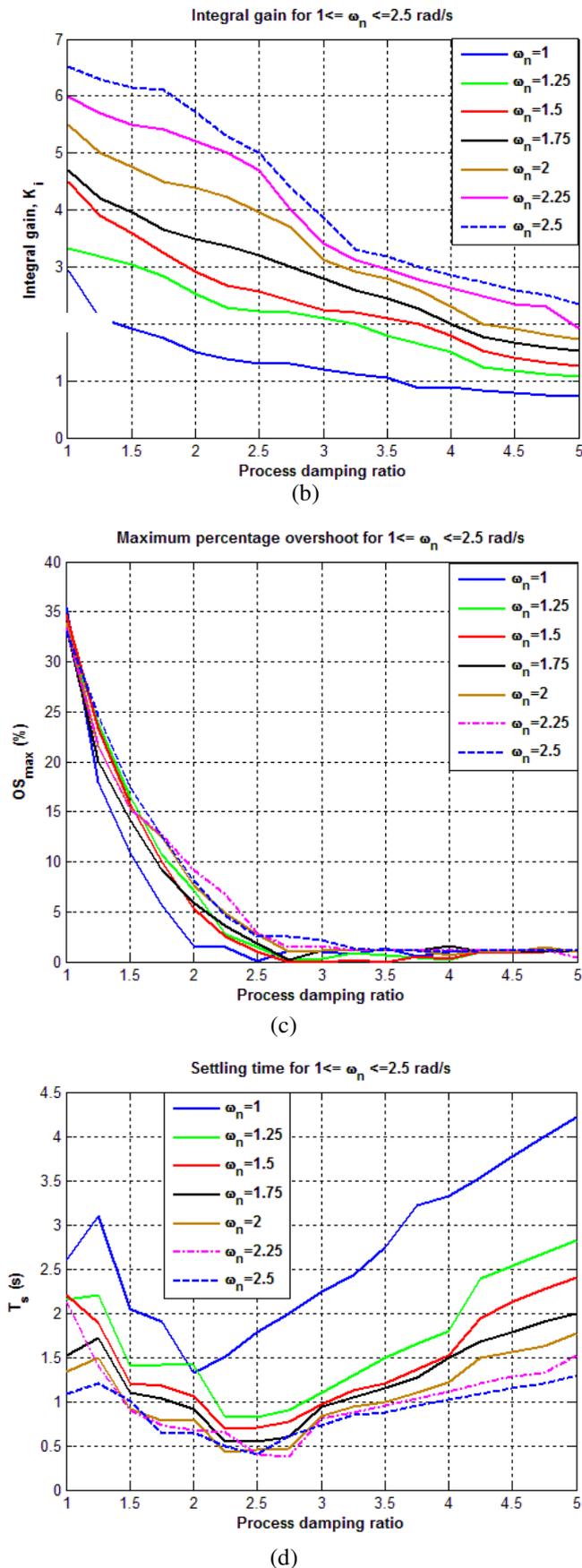


Fig.3 Tuning nomograms for $1 \leq \omega_n \leq 2.5$ rad/s.

VII. CASE STUDIES

The flexibility and simplicity of using the suggested nomograms are illustrated by two different case studies as follows:

Case study 1: Process of known parameters

A second-order-like process has a natural frequency of 0.75 rad/s and a damping ratio of 4.5. It is required to tune a PI controller used to control this process using the nomogram technique presented in this research paper.

The application of the nomogram technique is as follows:

1. In Fig.2(a) at a process damping ratio of 4.5 draw a vertical line to intersect the tuning curve corresponding to the 0.75 rad/s process natural frequency.
2. Draw a horizontal line to intersect the y-axis in the controller proportional gain as illustrated in Fig.2(a). The result is: $K_{pc} = 7.75$.
3. In Fig.2(b) at a process damping ratio of 4.5 draw a vertical line to intersect the tuning curve corresponding to the 0.75 rad/s process natural frequency.
4. Draw a horizontal line to intersect the y-axis in the controller integral gain as illustrated in Fig.2(b). The result is: $K_i = 0.75$.
5. In Fig.2(c) at a process damping ratio of 4.5 draw a vertical line to intersect the tuning curve corresponding to the 0.75 rad/s process natural frequency.
6. Draw a horizontal line to intersect the y-axis in the control system maximum percentage overshoot as illustrated in Fig.2(c). The result is: $OS_{max} = 1\%$.
7. In Fig.2(d) at a process damping ratio of 4.5 draw a vertical line to intersect the tuning curve corresponding to the 0.75 rad/s process natural frequency.
8. Draw a horizontal line to intersect the y-axis in the control system settling time as illustrated in Fig.2(d). The result is: $T_s = 4$ s.

The time response of the PI-controlled process using the present nomogram tuning technique is shown in Fig.4 for a unit step reference input.

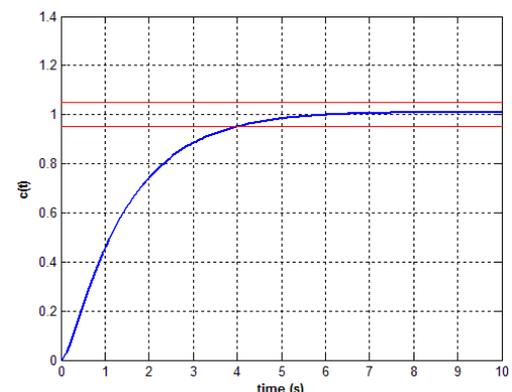




Fig.4 Time response of the control system in case study 1.

The performance characteristics of the PI-controlled process using Fig.4 are as follows:

- Maximum percentage overshoot: 0.907 % (compared with 1 % from the nomogram (10 % difference with the nomogram value).
- Settling time: 4 s (compared with 4 s from the nomogram (identical values).

The efficiency of the present approach is tested by comparison with PI-controller using the ITAE standard forms [22,23]. Using the ITAE standard forms, the PI controller parameters are estimated and given as:

$$K_{pc} = 84.95 \text{ and } K_i = 102.00$$

The time response of the PI controlled second order process using both tuning techniques is shown in Fig.5.

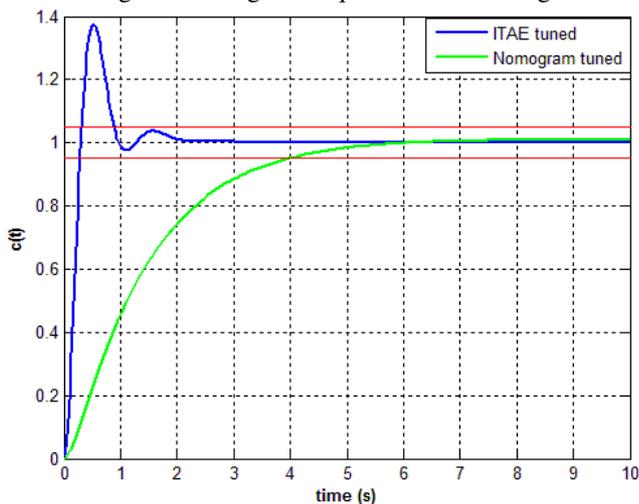


Fig.5 System step time response using two tuning techniques.

The control system has the time-based specifications:

- Maximum overshoot: 37.5 % compared with 0.907 % using the nomogram-tuning technique.
- Settling time: 1.85 s compared with 4 s using the nomogram-tuning technique.

Case study 2: Control system with desired specifications

This is another way of how to use the suggested tuning nomograms. Suppose that it is required to tune a PI controller and assign the transfer function of a second-order like process such that the closed-loop control system has no overshoot and a settling time less than one second. The procedure to achieve this goal is as follows:

1. Looking at Fig.3(c) we see that it is possible to get a closed-loop control system with zero overshoot when:
 - $\omega_n = 1$ and $\zeta = 0.5$.
 - $\omega_n = 1.5$ and ζ in the range: $2.75 \leq \zeta \leq 3.5$.
2. For $T_s < 1$ s, Fig.3(d) depicts the fact that all the

performance curves in Fig.3(d) can provide this condition for all the process natural frequency values in the figure except for the 1 rad/s value.

3. Combining the analysis in steps 1 and 2, we come the selection of the proper values of process natural frequency and damping ratio. That is:

$$\omega_n = 1.5 \text{ and } \zeta = 2.75$$

4. Now, drawing a vertical line in Fig.3(a) at $\zeta = 2.75$ intersects the performance curve corresponding to $\omega_n = 1.5$ in the value of the optimal proportional gain of the PI controller. That is: $K_{pc} = 10.2$ and $K_i = 2.4$.

5. To check the effectiveness of the nomogram-tuning technique use MATLAB to plot the time response of the closed-loop control system using Eq.3 and the process parameters assigned in step 3 and the PI controller parameters assigned in step 4. The result of this step is presented in Fig.6 with comparison with the tuning approach using the ITAE standard forms.

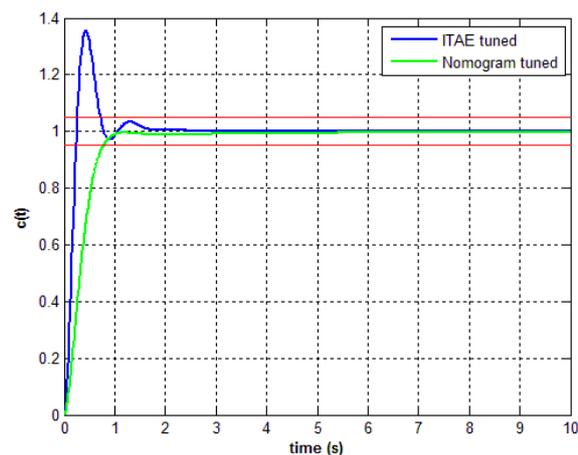


Fig.6 Step time response (case study 2).

The control system of the second case study has the time-based specifications:

- Maximum overshoot: 21.47 % using the ITAE standard forms compared with 0 % using the nomogram-tuning technique.
- Settling time: 1.49 s using the ITAE standard forms compared with 0.77 s using the nomogram-tuning technique.

The above analysis clarifies the effectiveness of the devised tuning technique without any need to tuning calculations or applying any optimization technique.

VIII. CONCLUSION

- A new tuning technique for PI controllers used to control second-order-like processes based on nomogram construction was presented.
- The construction of the nomograms was based on the application of unconstrained optimization technique to minimize an ISTSE objective function.



- The nomograms were construction using the process damping ratio and natural frequency.
- A procedure was given to tune a PI controller for a specific second-order-lime process having parameters in the range:

$$0.25 \leq \omega_n \leq 2.5 \quad \text{rad/s}$$
 And
$$1 \leq \zeta \leq 5.$$
- It was possible to go down with the maximum percentage overshoot of the closed-loop control system using a step input to zero (overshoot-free response).
- It was possible to go down with the settling time of the system time response to 0.4 second.
- When compared with another tuning technique, the present nomogram-based technique proved to be attractive and pioneer.
- The proposed technique did not require any calculations, but direct read from graphs providing the controller parameters and some of the time based specifications of the closed loop control system.

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