

# Selection of Best Transmit Structure Using Transmission Optimized Spatial Modulation

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**Abstract—** In this project select propose a best transmit structure using transmission optimized spatial modulation (SM), a unique single-stream multiple-input multiple-output (MIMO) transmission technique. SM enables a trade-off between the size of the spatial constellation diagram and the size of the signal constellation diagram. Based on this fact, the novel method, named transmission optimized spatial modulation (TOSM) minimizes the average bit error probability (ABEP). The traditional antenna selection methods, the proposed method relies on statistical channel state information (CSI) instead of instant CSI, and feedback is only needed for the optimal number of transmit antennas. The best transmit structure is selected based on circle packing method. Simulation results show that TOSM significantly improves the performance of SM at various channel correlations. TOSM offer better energy saving in both continuous and discontinuous transmission mode.

**Keywords—** Average bit Error Probability, Transmission optimized spatial modulation(TOSM), Multiple-Input Multiple-Output(MIMO), Circle packing.

## I. INTRODUCTION

Multi-antenna wireless communication systems have become popular due to their advantages of high throughput and enhanced reliability. Spatial modulation (SM) is a relatively new modulation technique which can resolve these issues by using only one transmit RF chain and choosing one antenna element in a multiple transmit antenna array and sending the information symbol through the chosen antenna [10],[11].

Two basic space modulation concepts exist in the literature. 1) Space Shift Keying (SSK) modulation [4], where the incoming bit stream is used to identify a single antenna of the antenna-array that is switched on for transmission. The information bits are mapped onto the channel impulse responses of the end-to-end wireless links. The main benefit of SSK modulation is a low implementation complexity. 2) Spatial

Modulation (SM) [11], which is a hybrid modulation scheme combining Phase Shift Keying (PSK) or Quadrature Amplitude Modulation (QAM) with SSK modulation. In SM, each block of information bits is transmitted through two information-carrying units: some bits are modulated using either PSK or QAM, while the others using SSK modulation. The main benefit of SM is a multiplexing gain, which is obtained with only a single active antenna.

While multi-stream MIMO schemes, such as vertical Bell Labs layered space-time (V-BLAST) and space-time block coding (STBC), offer high spectrum efficiency, unfortunately, they need multiple RF chains that heavily compromise the energy efficiency. Meanwhile, spatial modulation (SM) is a unique single-stream MIMO technique, where the bit stream is divided into blocks and each block is split into two parts: i) the first part activates one antenna from the antenna array while the remaining antennas do not emit a signal; ii) the bits in the second part are modulated by a signal constellation diagram, and sent out through the activated antenna. The use of a single active antenna makes SM a truly energy-efficient MIMO transmission technique, because only one RF chain is required, regardless of the number of transmit antennas used. At the same time, SM ensures spatial multiplexing gains as information is encoded in the antenna index. However, like all other MIMO schemes, SM suffers performance degradation caused by channel correlations. Trying to improve the performance of SM against channel variations, an adaptive method was proposed in, where one candidate is selected from several optional SM structures. Although the performance of SM can be improved to some extent, this method has the following weaknesses: i) it requires instant channel state information (CSI), and therefore it is not suitable for fast fading channels; ii) the relation between the adaptive selection and the channel correlation has not been

exploited; and iii) despite using a simplified modulation order selection criterion, it still requires significant processing power.

This paper proposes a novel adaptive antenna selection method for optimum transmission in SM. As a three dimensional modulation scheme, SM enables a trade-off between the size of the spatial constellation diagram and the size of the signal constellation diagram, while achieving the same spectrum efficiency. Based on this unique characteristic, transmission optimized spatial modulation (TOSM) aims to select the best combination of these two constellation sizes, which minimizes the average bit error probability (ABEP). To avoid the prohibitive complexity caused by exhaustive search, a two-stage optimization strategy is proposed. The first step is to determine the optimal number of transmit antennas, and this is performed at the receiver. In the second step, the required number of antennas is selected at the transmitter.

This paper remainder of the paper organized as follows section II discuss the existing method, section III discuss the system model including transmitter, channel and receiver model.

## II. EXISTING METHODS

### A. Space-Time Block Coding

Data is encoded using a space-time block code and the encoded data is split into  $n$  streams which are simultaneously transmitted using  $n$  transmit antennas. The received signal at each receive antenna is a linear superposition of the  $n$  transmitted signals perturbed by noise. Maximum likelihood decoding is achieved in a simple way through decoupling of the signals transmitted from different antennas rather than joint detection. This uses the orthogonal structure of the space-time block code and gives a maximum-likelihood decoding algorithm which is based only on linear processing at the receiver. Space-time block codes are designed to achieve the maximum diversity order for a given number of transmit and receive antennas subject to the constraint of having a simple decoding algorithm.

These codes achieve the maximum possible transmission rate for any number of transmit antennas using any arbitrary real constellation such as PAM. For an arbitrary complex constellation such as PSK and QAM, space-time block codes are designed that achieve 1/2 of the maximum possible transmission rate for any number of transmit antennas.

### B. Space Shift Keying

Space Shift Keying (SSK) modulation, where the incoming bit stream is used to identify a single antenna of the antenna-array that is switched on for transmission. The information bits are mapped onto the channel impulse responses of the end-to-end

wireless links. The main benefit of SSK modulation is a low implementation complexity. The diversity gain of SSK modulation is independent of the fading severity. The diversity gain of SSK is minimum.

### C. Spatial Modulation

Spatial modulation (SM) is a unique single-stream MIMO technique, where the bit stream is divided into blocks and each block is split into two parts: i) the first part activates one antenna from the antenna array while the remaining antennas do not emit a signal; ii) the bits in the second part are modulated by a signal constellation diagram, and sent out through the activated antenna. The use of a single active antenna makes SM a truly energy-efficient MIMO transmission technique, because only one RF chain is required, regardless of the number of transmit antennas used. At the same time, SM ensures spatial multiplexing gains as information is encoded in the antenna index. However, like all other MIMO schemes, SM suffers performance degradation caused by channel correlations. Trying to improve the performance of SM against channel variations, an adaptive method was proposed, where one candidate is selected from several optional SM structures.

Although the performance of SM can be improved to some extent, this method has the following weaknesses: i) it requires instant channel state information (CSI), and therefore it is not suitable for fast fading channels; ii) the relation between the adaptive selection and the channel correlation has not been exploited; and iii) despite using a simplified modulation order selection criterion, it still requires significant processing power.

### D. V-BLAST

Vertical Bell Labs layered space-time (V-BLAST) is a multi-stream MIMO schemes which offer high spectrum efficiency. In this method all the symbols of certain stream are transmitted through a single antenna (one stream per antenna). It eliminates space time wastage but losses transmit diversity, since each stream is tied to its antenna. Unfortunately, it needs multiple RF chains that heavily compromise the energy efficiency.

### E. Generalized Spatial Modulation

Generalized spatial modulation (GSM) is a relatively new modulation scheme for multi-antenna wireless communications. It is quite attractive because of its ability to work with less number of transmit RF chains compared to traditional spatial multiplexing (V-BLAST system). By using an optimum combination of number of transmit antennas ( $N_t$ ) and number of transmit RF chains ( $N_{rf}$ ), GSM can achieve better throughput and/or bit error rate (BER) than spatial multiplexing. A challenge

that arises in GSM with high rate is in the detection of the transmitted signal. Hence, there is a need for low complexity detection schemes with good performance for GSM with large number of transmit antenna elements.

### III SYSTEM MODEL

#### A. SM transmitter

The Fig.1 shows a  $N_t \times N_r$  SM-MIMO system transmitter, where  $N_t$  and  $N_r$  are the number of transmit antennas and the number of receive antennas, respectively. Unlike the original SM, only a subset of the transmit antennas is used. The size of the spatial constellation diagram, i.e. the number of utilized transmit antennas, is denoted by  $N$ , while the size of the signal constellation diagram is denoted by  $M$ . The bit stream is divided into blocks with the length of  $\eta_s$  bits, where  $\eta_s = \log_2(N) + \log_2(M)$  is the number of bits per symbol. Each block is then split into two units of  $\log_2(N)$  and  $\log_2(M)$  bits. The first part activates a single transmit antenna from the spatial constellation diagram, and the currently active antenna is denoted by  $x_{tact}$ . The second part chooses the corresponding symbol  $X_l (1 \leq l \leq M)$  from a specific signal constellation diagram, such as phase shift keying (PSK) or quadrature amplitude modulation (QAM), and sends it out through the activated antenna. The transmitted signal of SM is represented by a vector  $\mathbf{x} = [0, \dots, x_{tact}, \dots, 0]$  of  $N$  elements, where the  $tact$ -th element is  $X_l$  and all other elements are zero.

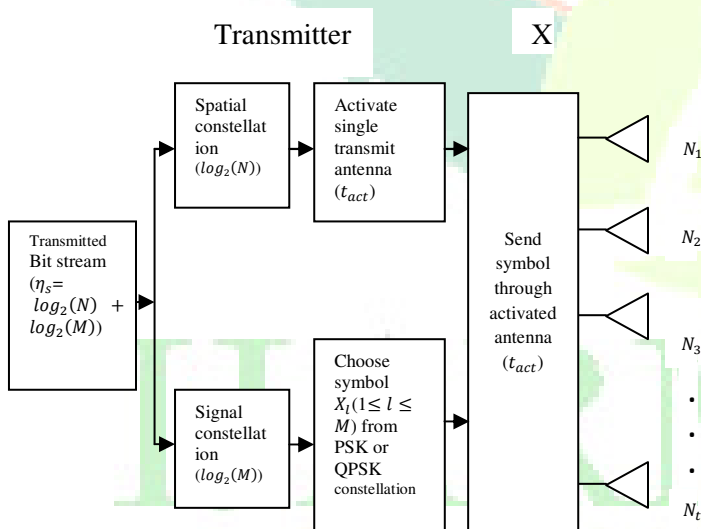


Fig 1. Block diagram of SM transmitter

#### B. SM wireless channel

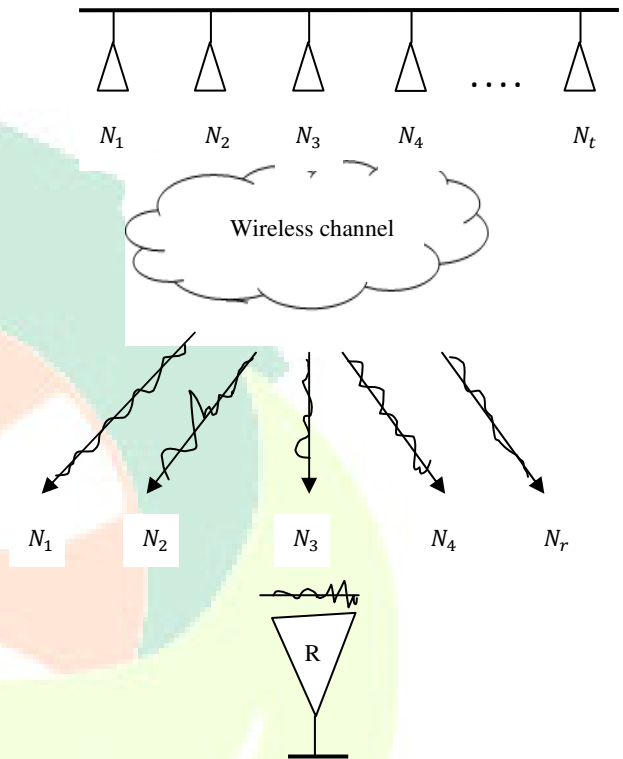


Fig. 2 SM wireless channel

The fading coefficient of the link from the  $t$ -th transmit antenna to the  $r$ -th receive antenna is denoted by  $h_{t,r} = \beta_{t,r} \exp(j\phi_{t,r})$ , where  $\beta_{t,r}$  and  $\phi_{t,r}$  are the amplitude and the phase, respectively. The channel fading distribution as well as the CSI is assumed to be known at the receiver. Nakagami- $m$  fading is considered in this paper, i.e.  $\beta_{t,r} \sim \text{Nakagami}(m_{t,r}, \Omega_{t,r})$ , where  $m_{t,r}$  is the shape parameter (when  $m_{t,r} = 1$ , the channel is Rayleigh fading) and  $\Omega_{t,r}$  is the spread controlling parameter. The phase  $\phi_{t,r}$  is uniformly distributed between  $(-\pi, \pi]$ .

This paper focus on selecting the transmit antennas, the receive antennas are assumed to be independent without loss of the generality. The correlation coefficient between the amplitudes of the two propagation paths from the transmit antennas  $t_i$  and  $t_j$  to the  $r$ -th receive antenna is denoted by  $\rho_{t_i,t_j,r}$ . The exponential correlation matrix is based on the fact that the channel correlation decreases with increasing the distance between antennas. The absolute distance between  $t_i$  and  $t_j$  is denoted by  $d_{t_i,t_j}$ , and the correlation between those two antennas is given by



$$p_{av} = \frac{1}{N_r} \left( \frac{1}{N_t(N_t-1)} \sum_{t_i=1}^{N_t} \sum_{t_j \neq t_i=1}^{N_t} p_{t_i, t_j, r} \right) \quad (1)$$

The average degree of the channel correlations, denoted by  $\rho_{av}$ , is calculated by:

$$p_{t_i, t_j, r} = p_{s(r)}^{d_{t_i, t_j}}, 0 \leq p_{s(r)} \leq 1 \quad (2)$$

#### C. SM receiver

Fig .3 shows the received signal is  $y = HX + w$ , where  $H$  stands for the channel matrix, the vector  $w = [w_1, w_1, \dots, w_{N_r}]^T$  and  $w_r$ , the noise at the  $r$ -th receive antenna, is a sample of complex additive white Gaussian noise with distribution  $CN(0, N_0)$ . Across receive antennas, the noise components are statistically independent. The signal-to-noise ratio (SNR) is defined as  $\gamma = E_m L / N_0$ , where  $E_m$  is the average energy per symbol transmission and  $L$  denotes the path loss without shadowing. In addition, the required RF output energy per bit is denoted by  $E_b = E_m / \eta_s$ . The transmitted information bits are decoded by the joint maximum likelihood (ML) detection.

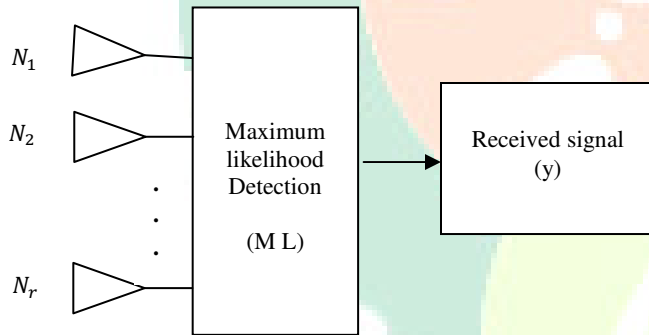


Fig .3 SM Receivers

#### IV. TOSMOVER GENERALIZED FADING CHANNELS

##### A. ABEP FOR TOSM

For correlated and identically distributed (c.i.d.) Rayleigh fading channels, have  $m_r = 1$  and  $\Omega_{t,r} = \Omega$  for all  $t$  and  $r$ . The ABEP is given by

$$ABEP = \frac{B(M)^{2Nr} + C(2^{2\eta_s} \eta_s - M \log_2(M))}{\eta_s \gamma^{Nr}} \quad (3)$$

where ,

$$B = \left( \frac{2}{\Omega} \right)^{Nr} \frac{1}{\pi^{2Nr+1}} \int_0^\pi (\sin \theta)^{2Nr} d\theta \quad (4)$$

And

$$C = \frac{4^{Nr-1} \Gamma(N_r + 0.5)}{\Omega^{Nr} \sqrt{\pi} \Gamma(N_r + 1)} \left( \sum_{k=0}^{-\infty} \frac{\Gamma(2k+1) \rho_{av}^k}{4^k (k!) \Gamma(k+1)} \right) \quad (5)$$

##### B. BASE STATION ENERGY CONSUMPTION BASED ON TSOM

The required  $E_m$  using TOSM is computed by,

$$E_m = \frac{N_0}{L} \left( \frac{F(M_{opt})}{\eta_s R_b} \right)^{\frac{1}{Nr}} \quad (6)$$

where  $F(M) = B(M)^{2Nr} + C(2^{2\eta_s} \eta_s - M \log_2(M))$  and  $R_b$  denotes the value of the target BER.

The required RF output power is obtained by,

$$P_m = \frac{R_b N_0}{\eta_s L} \left( \frac{F(M_{opt})}{\eta_s R_b} \right)^{\frac{1}{Nr}} \quad (7)$$

##### C. CONTINUOUS MODE

In the continuous mode, the RF chains are always delivering output power of the same level. The energy consumption per bit of a BS based on TOSM is obtained by with  $N_{act} = 1$

$$E_{BS} = \frac{P_0}{R_b} + \frac{N_0}{\eta_s L} \left( \frac{F(M_{opt})}{\eta_s R_b} \right)^{\frac{1}{Nr}} \quad (8)$$

Where,  $\square$  stands for the slope that quantifies the load dependence  $N_{act}$  activated antennas.

##### D. DTX MODE

The DTX mode conveys data with full load, and the instantaneous data rate. Compute  $E_{BS}$  in the DTX mode as follows,

$$E_{BS} = \frac{P_0}{R_b} + \frac{N_0}{\eta_s L} \left( \square + \frac{P_0 - P_s}{P_{max}} \right) \left( \frac{F(M_{opt})}{\eta_s R_b} \right)^{\frac{1}{Nr}} \quad (9)$$

##### E. DIRECT ANTENNA SELECTION

Select a subarray of  $N_{opt}$  antennas from the size- $N_t$  antenna array. The chosen subset should achieve the minimum ABEP of all subarrays with the same size. Since  $B_N$  is irrelevant to the channel correlations, the problem is equivalent to finding the subarray with a minimum  $c_N$ . Like the traditional transmit antenna selection (TAS) methods, this issue can be solved by an exhaustive search. However, this results in an unaffordable complexity for a large  $\eta_s$ . Taking  $\eta_s = 6$  and  $N_{opt} = 16$  as an example, the full search space is about  $5 \times 1014$ , which is prohibitive for practical implementations. Here, propose a novel TAS method based on circle packing, which can directly determine the selection. As the correlation coefficient  $p_{t_i, t_j}$  is inversely proportional to the distance  $d_{t_i, t_j}$ , a rational solution is to maximize the minimum geometric distance between any pair of the chosen antennas. This is equivalent to the circle packing problem in mathematics.

Fig 4. shows the circle packing solutions for various numbers of antennas, where the antennas are located at the circle centers. In the original problem, each circle must fit inside the square boundary. The problem at hand is slightly different where the circle centers are restricted to be inside the boundary, and in

Fig.4 this is shown by dashed lines. It is worth noting that this solution requires fully flexible positions. Thus, we refer to it as ideal circle packing (ICP). However, the antenna positions are fixed in practice, and the subarray cannot be perfectly allocated by ICP. Instead, a realistic circle packing (RCP) is developed by selecting those antennas closest to the ideal positions. In Fig 5, an RCP solution is demonstrated for the case of  $N_t = 32$  and  $N = 8$ . As can be observed, the selection presents a similarity to the solution for  $N = 8$  in Fig.4. With an increase of  $N_t$ , the RCP solution becomes closer to ICP as the antenna array supplies a larger flexibility in positions.

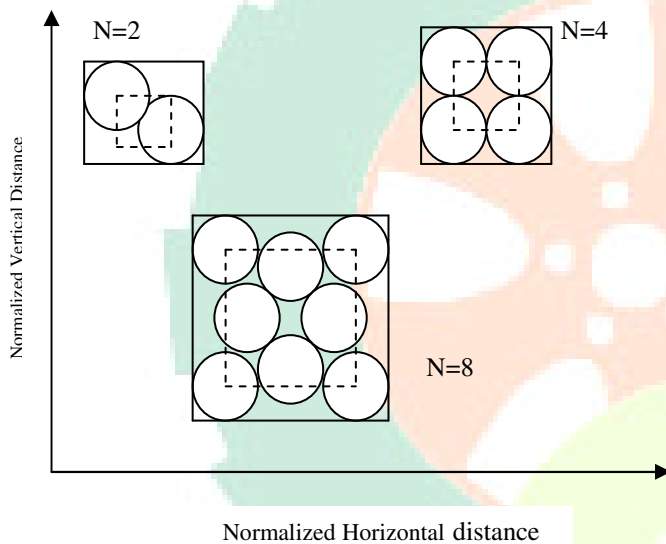


Fig 4. Examples of circle packing problems

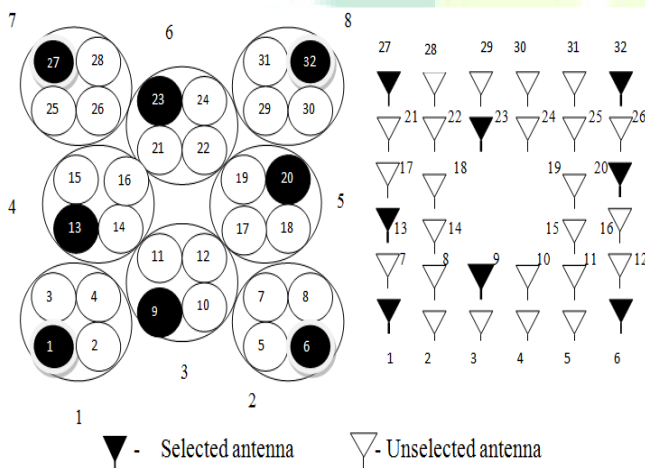


Fig. 5 RCP for selecting 8 out of 32 antennas

#### IV. RESULT AND DISCUSSION

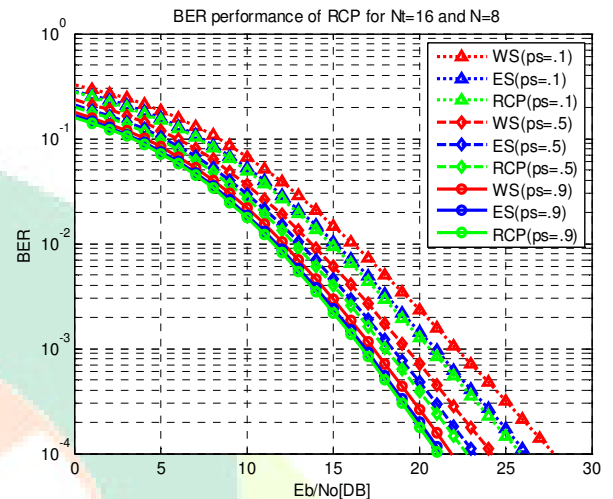


Fig 6 BER performance of RCP  $N_t=16$  and  $N=8$

The BER performance of the proposed RCP approach is evaluated against two baseline schemes: i) the exhaustive search (ES); and ii) the worst case where the neighbouring antennas are selected. We refer to this scheme as worst selection (WS) in the sequel. Fig .6 and Fig .7 present the BER performance of RCP for  $\eta_s = 4$  and 5, respectively. Due to the intractable complexity of ES, the results when  $\eta_s > 5$  are not presented. In addition, the antenna area is assumed to be the same to ensure a fair comparison for different  $\eta_s$ . Therefore,  $\rho_s$  is used instead of  $\rho_{av}$ . As shown, the RCP scheme achieves almost the same performance as ES with a gap of less than 0.3 dB. Furthermore, the negligible difference between RCP and ES is barely affected by the channel correlations, whereas the performance of WS becomes much worse as the correlation increases. To achieve the same BER value of  $1 \times 10^{-4}$  in the case of selecting 8 out of 32 antennas, in comparison with WS, RCP obtains an energy saving of 1.1 dB and 2.0 dB at  $\rho_s = 0.1$  and 0.9, respectively.

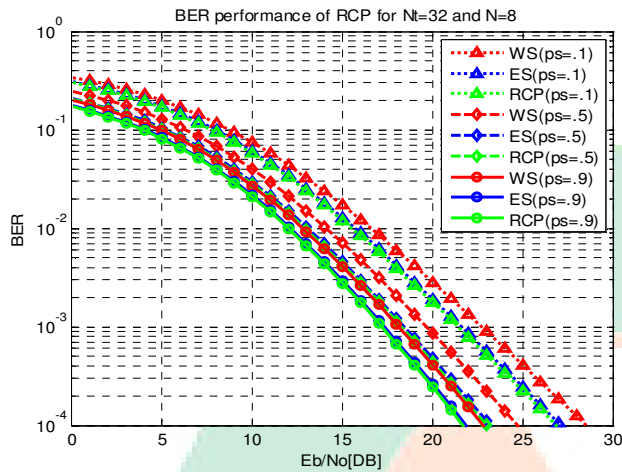


Fig. 7 BER performance of RCP  $N_t=32$  and  $N=8$

The complexity of the MIMO system depends on the number of RF chains rather than the total number of transmit antennas. Despite the requirement of large antennas at the transmitter, TOSM needs only one RF chain. For this reason, it is reasonable to compare our approach to fixed-SM schemes with the same  $\eta_s$ . Based on the obtained optimal  $N$ , we evaluate the BER performance of TOSM. Assuming  $N_r = 2$  and  $E_b/N_0 = 25$  dB, Figs 8-10 show the BER results against the channel correlation for  $\eta_s = 4, 5$  and  $6$ , respectively. The case of  $N = 1$  is referred to as single-input multiple-output (SIMO).

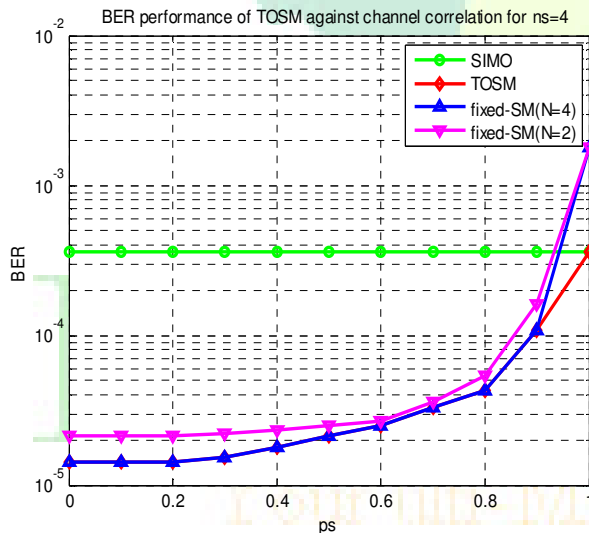


Fig. 8 BER performance of TOSM against channel correlation for  $\eta_s = 4$ .

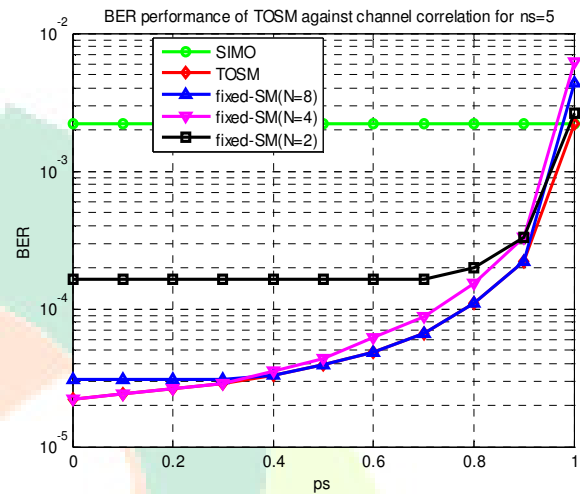


Fig. 9 BER performance of TOSM against channel correlation for  $\eta_s = 5$ .

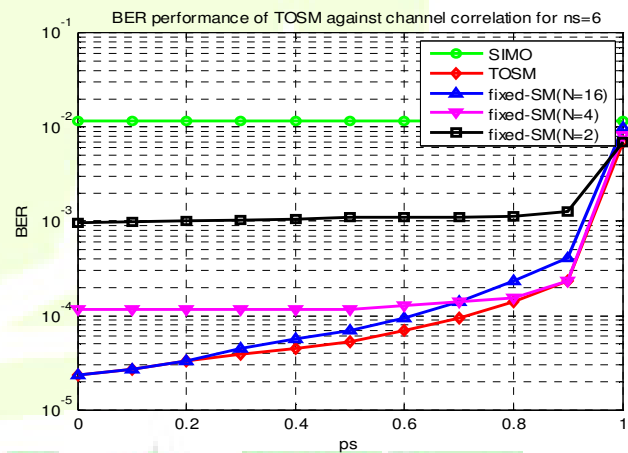


Fig. 10 BER performance of TOSM against channel correlation for  $\eta_s = 6$ .

The following trends are observed: i) fixed-SM with more antennas is not always better than those using fewer antennas. This signifies that the benefit does not simply come from employing more transmit antennas; ii) TOSM always performs better than or equal to fixed-SM schemes, which validates the optimization results; and iii) when  $\eta_s$  increases, TOSM employs more transmit antennas and performs much better than the fixed-

SM with a small  $N$ . Specifically, TOSM slightly outperforms fixed-SM with  $N = 2$  at both low and high correlations for  $\eta_s = 4$ . However, for  $\eta_s = 5$  and 6, TOSM can always achieve a significant gain except when the channel correlation is extremely high. Similar, but less pronounced trends are noticed at lower SNRs

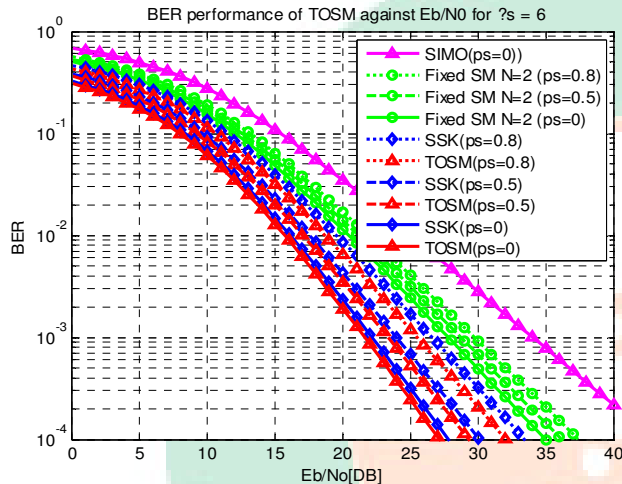


Fig. 11 BER performance of TOSM against  $E_b/N_0$  for  $\eta_s = 6$

In Fig. 11, the BER performance of TOSM is shown as a function of  $E_b/N_0$  for  $\eta_s = 6$ . As can be seen, TOSM significantly outperforms the other schemes for all presented SNRs and various channel correlation degrees. When the channels are independent, i.e.  $ps = 0$ , TOSM saves energy in the regions of 0.8 dB, 8.7 dB, and 15.1 dB relative to SSK, fixed-SM with  $N = 2$ , and SIMO, respectively. As  $ps$  increases, TOSM outperforms SSK more significantly. Conversely, fixed-SM with  $N = 2$  is only slightly affected by the channel correlation, and the advantage of TOSM is diminishing with an increase of  $ps$ . However, the gain of TOSM over fixed-SM with  $N = 2$  still exceeds 4 dB at  $ps = 0.8$ .

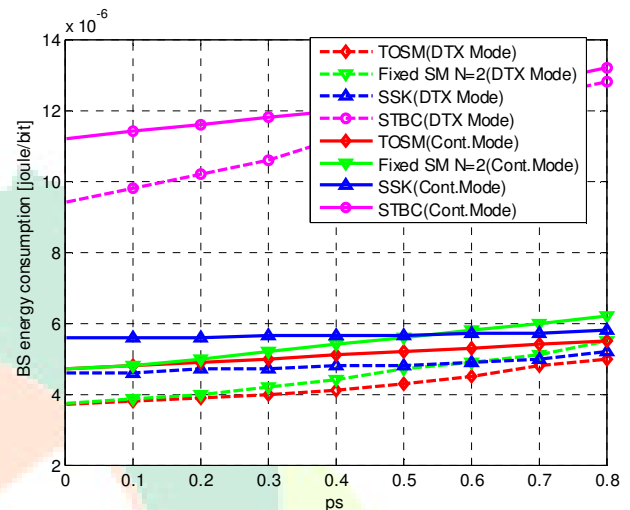


Fig. 12 BS energy consumption of TOSM for  $R_b = 30$  Mbit/s

The overall BS energy consumptions in both the continuous mode and the DTX mode are shown in Fig. 12. Compare to all other methods TOSM has better performance. TOSM save more energy in the DTX mode compare to continuous mode.

## V. CONCLUSION

In this project proposed an optimum transmit structure for SM, which balances the size of the spatial constellation diagram and the size of the signal constellation diagram. Instead of using exhaustive search, a novel two-stage TAS method has been proposed for reducing the computational complexity, where the optimal number of transmits antennas and the specific antenna positions are determined separately. The first step is to obtain the optimal number of transmit antennas by minimizing a simplified ABEP bound for SM. In the second step, a direct antenna selection method, named RCP, was developed to select the required number of transmit antennas from an antenna array. Results show that TOSM improves the BER performance of the original SM significantly.

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